PSGVP MANDAL'S

SHRI S I PATIL ARTS, G B PATEL SCIENCE & STKVS COMMERCE COLLEGE, SHAHADA, DIST-NANDURBAR

DEPARTMENT OF MATHEMATICS & STATISTICS

PROGRAME NAME: CERTIFICATE COURSE IN MATHEMATICS IN ECONOMICS

PROGRAM DESCRIPTION:

This program is aimed at introducing the fundamentals of basic mathematics & its applications in economics to undergraduate as well as post-graduate students. The background expected includes a prior knowledge of mathematics from HSC. The goals of the program to understand the basic concepts of mathematics and their applications in economics. Also, to understand the application of mathematics in economical models.

General Objectives:

To apply the knowledge applications of mathematics in economics.

PROGRAM OUTCOMES:

PO1: Scientific and technical knowledge will be developed in students.

PO2: Students will obtain basic practical skills.

PO3: Students will become employable.

PO4: Students will possess basic subject knowledge for higher studies.

PO5: Students will be aware of and able to develop solution oriented approach towards various economic issues.

PROGRAM SPECIFIC OUTCOMES:

PSO1: A students should be able to recall basic facts about mathematics & economics.

PSO2: A students should get suitable experience to global and local concerns that discover them many aspects of Mathematics & economics.

PSO3: A student is prepared with mathematical & economical modelling ability, problems solving skills & communication skills for various kinds of employment.

PSO4: A students should be able to apply their skills & techniques in order to process the relevant conclusions.

PSO5: Making students to develop a positive attitude towards mathematics & economics as valuable subject of study.

DURATION OF THE PROGRAM: **1 Year**

PROGRAM STRUCTURE:

Sr. No.	Paper	Name	Credits
1	Paper-I	Mathematical Economic-I	6
2	Paper-II	Mathematical Economic-II	6
3	Paper-III	Practical Course	8

ELLIGIBILITY: **H.S.C. Passed**

FEES: **2000/-**

MODE OF EXAMINATION: ANNUAL (Online/offline)

PAPER-I: MATHEMATICAL ECONOMICS-I

COURSE OUTLINE

MATHEMATICAL ECONOMICS-I	**ME-I**	**CCME-101**
Course Title	Short Title	Course Code

Course Details:

Hours/Week	No. of Weeks	Total Hours	Credits
03	30	90	6

Prerequisite Course: 12th (Science or Commerce faculty).

Course Outcomes:

After successful completion of this course the student will be able to:

CO1: learn basic concepts of mathematics & its application in economics.

CO2: learn the limits & derivatives.

CO3: learn the economics applications of derivative.

CO4: learn the concepts of multivariable functions.

CO5: learn the calculus of multivariable functions in economics.

CO6: learn the applications of exponential & logarithmic function in economics.

CO-PO MAPPING:

PROGRAM OUTCOMES					
GRADUATE ATTRIBUTES	Critical thinking	Problem Solving	Self-directed learning	Lifelong learning	Disciplinary Knowledge
Course Objectives	PO1	PO2	PO3	PO4	PO5
CO1	2	3	3	3	3
CO2	2	3	1	3	2
CO3	3	3	3	3	2
CO4	1	3	1	3	2
CO5	1	3	3	3	2
CO6	1	3	3	3	3
Average	1.66	3	2.33	3	2.33
PROGRAM SPECIFIC OUTCOMES					
Course Objectives	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	2	-	-	2
CO2	3	3	1	1	2
CO3	1	3	3	2	3
CO4	1	1	-	1	2
CO5	-	3	3	2	2
CO6	-	3	3	2	2
Average	2	2.66	2.5	2	2.16

Mapping Corelation	Low	Medium	High	No
	1	2	3	-

Attainment

CO1	CO2	CO3	CO4	CO5	CO6
21	**21**	**26**	**15**	**22**	**23**

CONTENT:

CHAPTER-I BASIC MATHEMATICS & APPLICATIONS IN ECONOMICS **Hours = 15**

- Exponents
- Polynomials
- Equations: Linear & Quadratic
- Simultaneous Equations
- Functions
- Graphs, Slopes & Intercepts
- Isocost Lines
- Supply and Demand Analysis
- Income Determination Models
- *IS-LM* Analysis

CHAPTER-II LIMITS & DERIVATIVE **Hours = 15**

- Limits
- Continuity
- The Slope of a Curvilinear Function
- The Derivative
- Differentiability and Continuity
- Derivative Notation
- Rules of Differentiation
- Higher-Order Derivatives
- Implicit Differentiation

CHAPTER-III ECONOMIC APPLICATIONS OF DERIVATIVE **Hours = 15**

- Increasing and Decreasing Functions
- Concavity and Convexity
- Relative Extrema
- Inflection Points
- Optimization of Functions
- Successive-Derivative Test for Optimization
- Marginal Concepts
- Optimizing Economic Functions
- Relationship among Total, Marginal, and Average Concepts

CHAPTER-IV MULTIVARIABLE FUNCTIONS **Hours = 15**

- Functions of Several Variables and Partial Derivatives

- Rules of Partial Differentiation
- Second-Order Partial Derivatives
- Optimization of Multivariable Functions
- Constrained Optimization with Lagrange Multipliers
- Significance of the Lagrange Multiplier
- Differentials
- Total and Partial Differentials
- Total Derivatives
- Implicit and Inverse Function Rules

CHAPTER-V CALCULUS OF MULTIVARIABLE FUNCTIONS IN ECONOMICS Hours=15

- Marginal Productivity
- Income Determination Multipliers and Comparative Statics
- Income and Cross Price Elasticities of Demand
- Differentials and Incremental Changes
- Optimization of Multivariable Functions in Economics
- Constrained Optimization of Multivariable Functions in Economics
- Homogeneous Production Functions
- Returns to Scale
- Optimization of Cobb-Douglas Production Functions
- Optimization of Constant Elasticity of Substitution Production Functions

CHAPTER-VI EXPONENTIAL AND LOGARITHMIC FUNCTIONS IN ECONOMICS Hours=15

- Exponential Functions & Logarithmic Functions
- Logarithmic Transformation of Nonlinear Functions
- Interest Compounding
- Effective vs. Nominal Rates of Interest
- Discounting
- Converting Exponential to Natural Exponential Functions
- Estimating Growth Rates from Data Points
- Alternative Measures of Growth
- Optimal Timing
- Derivation of a Cobb-Douglas Demand Function Using a Logarithmic Transformation

REFERENCES:

1. Schaum's Outline of Theory & Problems of Introduction to Mathematical Economics, Third edition by Edward T. Dowling, Ph.D.

2. Basic Mathematics for Economics, Second edition by Mike Rosser.

3. An Introduction to Mathematics for Economics by Akihito Asano.

4. Mathematical Analysis by S. C. Malik.

5. Elements of the Differential and Integral Calculus with applications by William Shaffer Hall.

PAPER-II: MATHEMATICAL ECONOMICS-II

COURSE OUTLINE

MATHEMATICAL ECONOMICS-II	**ME-II**	**CCME-102**
Course Title	Short Title	Course Code

Course Details:

Hours/Week	No. of Weeks	Total Hours	Credits
03	30	90	6

Prerequisite Course: 12th (Science or Commerce faculty).

Course Outcomes:

After successful completion of this course the student will be able to:

CO1: learn basic concepts of matrix.

CO2: learn the applications of matrix in economics.

CO3: learn the comparative statics.

CO4: learn the concepts of integral calculus and applications in economics.

CO5: learn the first order differential equations and applications in economics.

CO6: learn the first order difference equations and applications in economics.

CO-PO MAPPING:

PROGRAM OUTCOMES					
GRADUATE ATTRIBUTES	Critical thinking	Problem Solving	Self-directed learning	Lifelong learning	Disciplinary Knowledge
Course Objectives	PO1	PO2	PO3	PO4	PO5
CO1	1	3	1	3	2
CO2	2	3	3	3	3
CO3	3	3	3	3	2
CO4	3	3	3	3	2
CO5	2	3	3	3	2
CO6	2	3	3	3	3
Average	2.16	3	2.66	3	2.33
PROGRAM SPECIFIC OUTCOMES					
Course Objectives	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	1	-	-	2
CO2	3	3	3	2	3
CO3	1	3	3	3	3
CO4	2	2	2	2	2
CO5	2	3	3	2	3
CO6	2	3	3	2	3
Average	2.16	2.5	2.33	1.83	2.66

Mapping Corelation	Low	Medium	High	No
	1	2	3	-

Attainment

CO1	CO2	CO3	CO4	CO5	CO6
16	**28**	**27**	**24**	**26**	**27**

CONTENT:

CHAPTER-I MATRIX ALGEBRA **Hours=15**

- Introduction
- Definitions and Terms, Addition and Subtraction of Matrices
- Scalar Multiplication & Vector Multiplication
- Multiplication of Matrices
- Identity and Null Matrices
- Matrix Expression of a System of Linear Equations
- Determinants & Uses in economics
- Cramer's Rule for Matrix Solutions

CHAPTER-II MATRIX APPLICATIONS IN ECONOMICS **Hours=15**

- Inverse of Matrix
- Jacobian
- Hessian
- Discriminant
- Higher-Order Hessians
- The Bordered Hessian for Constrained Optimization
- Input-Output Analysis.
- Eigenvalues & Eigenvectors

CHAPTER-III COMPARATIVE STATICS **Hours=15**

- Introduction to Comparative Statics
- Comparative Statics with One Endogenous Variable
- Comparative Statics with More Than One Endogenous Variable
- Comparative Statics for Optimization Problems
- Comparative Statics Used in Constrained Optimization
- The Envelope Theorem

CHAPTER-IV INTEGRAL CALCULUS **Hours=15**

- Integration, Rules of Integration
- Initial Conditions and Boundary Conditions
- Integration by Substitution
- Integration by Parts
- Economic Applications

- The Definite Integral & Properties
- The Fundamental Theorem of Calculus
- Area Under a Curve & Between Curves
- Improper Integrals
- L'Hospital's Rule
- Consumers' and Producers' Surplus
- The Definite Integral and Probability

CHAPTER-V FIRST-ORDER DIFFERENTIAL EQUATIONS **Hours=15**

- Definitions and Concepts
- General Formula for First-Order Linear Differential Equations
- Exact Differential Equations and Partial Integration
- Integrating Factors
- Rules for the Integrating Factor
- Separation of Variables
- Economic Applications
- Phase Diagrams for Differential Equations

CHAPTER-VI FIRST-ORDER DIFFERENCE EQUATION **Hours=15**

- Definitions and Concepts.
- General Formula for First-Order Linear Difference Equations
- Stability Conditions
- Lagged Income Determination Model
- The Cobweb Model
- The Harrod Model
- Phase Diagrams for Difference Equations

REFERENCES:

1. Schaum's Outline of Theory & Problems of Introduction to Mathematical Economics, Third edition by Edward T. Dowling, Ph.D.

2. Basic Mathematics for Economics, Second edition by Mike Rosser.

3. An Introduction to Mathematics for Economics by Akihito Asano.

4. Mathematical Analysis by S. C. Malik.

5. Elements of the Differential and Integral Calculus with applications by William Shaffer Hall.

6. Ordinary and Partial Differential equation by M. D. Raisinghania.

7. A Textbook of Matrices by Shanti Narayan & Dr. P. K. Mittal.

PAPER-III: PRACTICAL COURSE

COURSE OUTLINE

PRACTICAL COURSE	**PC**	**CCME-103**
Course Title	Short Title	Course Code

Course Details:

Hours/Week	No. of Weeks	Total Hours	Credits
04	30	120	8

Sr. No.	Unit	Topics	Hours
1	Chapter-I	EXAMPLES ON CHAPTER-I OF PAPER-I & PAPER-II	20
2	Chapter-II	EXAMPLES ON CHAPTER-II OF PAPER-I & PAPER-II	20
3	Chapter-III	EXAMPLES ON CHAPTER-III OF PAPER-I & PAPER-II	20
4	Chapter-IV	EXAMPLES ON CHAPTER-IV OF PAPER-I & PAPER-II	20
5	Chapter-V	EXAMPLES ON CHAPTER-V OF PAPER-I & PAPER-II	20
6	Chapter-VI	EXAMPLES ON CHAPTER-VI OF PAPER-I & PAPER-II	20

LIST OF PRACTICALS

Practical-I

1. Solve

a. $(30x - 12y) + (42x - 64y)$
b. $(43x + 23y) + (42x - 60y)$
c. $(29x - 17y) + (15x - 54y)$
d$(3x - 2y) + 8y.$
e. $(3x^2 - 5x - 56) + (2x^2 - 64)$
f. $(7x^2 - 2x - 6) + (2x^2 + 6x - 6)$

2. Solve

a. $(x^2 - 5)(x^2 + 4)$
b. $(2x^2 - 3x)(5x^2 + 2)$
c. $(9x - 7y)(4x - 6y)$
d. $(x^2 - 2x - 6)(x^2 + 6x - 6)$
e. $(x - 12y) + (42x - y)$
f. $(3x - 2y)y^2$

3. Solve

a. $4(3x - 2) = 8x$
b. $4(3x + 2) - 2 = 6x - 4$
c. $\frac{x}{3} - 12 = \frac{x}{12} + 15$
d. $\frac{5}{x} + \frac{3}{x+4} = \frac{7}{x}$

4. Find $f(-4), f(2), f(-3), f(1.2)$ for each of the following function

a. $f(x) = 7x^2 + 16x - 6$
b. $f(x) = x^2 - 7x + 3$

5. Graph the following equation and indicate their respective slopes & intercepts

a. $3y + 15x = 30$
b. $2y - 6x = 12$

Practical-2

1. A complete demand function given by the equation $Q_d = -25P + 0.02Y + 2P_r + 4T$, where P is the price of the good, Y is income, P_r is the price of a related good and T is taste. Draw the graph for the demand function assuming $Y = 5000, P_r = 25, T = 30$.

2. A person has $100 to spend on two goods (X, Y) whose respective prices are $4 & $6

a. Draw a budget line showing all the different combinations of the two goods that can be bought with the given budget(B).

b. What happens to the original budget line, If the budget falls by 20 percent?

c. What happens to the original budget line, If the price of X doubles?

d. What happens to the original budget line, If the price of Y falls to 5?

3. Given $Y = C + I, C = 40 + 0.6Y$ & $I_0 = 40$.

a. Graph the consumption function.

b. Graph the aggregate demand function, $C + I_0$.

c. Find the equilibrium level of income from the graph.

4. Use a graph to show the addition of a lump-sum tax influences the parameters of the income determination model.

$$Y = C + I$$

$$C = 90 + 0.5Y$$

$$I_0 = 40$$

5. Use a graph to show the addition of a lump-sum tax influences the parameters of the income determination model.

$$Y = C + I$$

$$C = 90 + 0.5Yd$$

$$I_0 = 40$$

Practical-3

1. Find a. $\lim_{x\to 2}[x^2(x-1)]$. b. $\lim_{x\to 2}[5x^2 - 2x - 1]$

c. $\lim_{x\to 4}\frac{2x^2-10x-1}{4x}$ d. $\lim_{x\to 3}\frac{x-1}{2x^2-2}$

2. Whether the function $f(x) = \frac{x-3}{x^2-9}$ at $x = 3$, is continuous.

3. Differentiate the following w. r. t. x

a. $y = \frac{x^2-2x-1}{4x+3}$ b. $y = \frac{x^2-1}{(x^2+3)^2}$

4. For each of the following functions, investigate the successive derivatives and evaluate them at $x = 2$. a. $y = x^3 + 4x^2 + 2x - 4$ b. $y = (4x - 2)(x + 2)$

5. Use implicit differentiation to find $\frac{dy}{dx}$

a. $2x^2 - y^2 = 43$

b. $2x^4 + x^3 + 3y^5 = 132$

Practical-4

1. Use implicit differentiation to find $\frac{dy}{dx}$

a. x^6y^5

b. $3x^2 + 4xy + y^2 = 56$

2. Find $\frac{dy}{dx}$ for $4x^4 + x^3y + 2y^2 = 45$

3. Find $\frac{d^2y}{dx^2}$ for $2x^2 - y^2 = 43$ at $x = 1$.

4. Find $\frac{d^2y}{dx^2}$ for $x^2y + 3x + y^2 = 6$ at $x = 4$.

5. Find $\frac{d^2y}{dx^2}$ for $f(x) = (x^4 - 3)(x^3 - 2)$ at $x = 2$.

Practical-5

1. Test whether the following functions are increasing, decreasing or stationary at $x = 2$.

a. $y = 2x^2 - 4x + 2$

b. $y = x^3 - 5x^2 + x - 4$

2. Test whether the following functions are concave or convex at $x = 1$.

a. $y = 3x^3 - 2x^2 + 5x - 8$

b. $y = (3x^2 - 7)^2$

3. Find the relative extrema for the following functions by finding the critical value(s) &determining if at critical value(s) the function is at relative maximum or minimum.

a. $f(x) = -5x^2 + 12x + 100x - 35$

b. $f(x) = 2x^3 - 20x^2 + 120x - 24$

4. For the following functions find the critical value(s) & test to see if at critical value(s) the function is at a relative maximum, minimum or possible infection point.

a. $y = -(x - 4)^2$

5. For the following functions find the critical value(s) & test to see if at critical value(s) the function is at a relative maximum, minimum or possible infection point.

a. $y = (3 - x)^3$

Practical-6

1. Find the marginal and the average functions for $TC = 3Q^2 + 7Q + 12$, evaluate it at $Q = 3, Q = 5$.

2. Find the marginal expenditure functions associated with supply function $P = Q^2 + 2Q + 1$, Evaluate it at $Q = 4, Q = 10$.

3. Find the MR function for demand function $44 - 4P - Q = 0$ and evaluate at $Q = 4, Q = 10$.

4. Faced with two distinct demand functions

$$Q_1 = 24 - 0.2P_1 \, , Q_2 = 10 - 0.05P_2$$

where $TC = 35 + 40Q$, what price will the firm charge with discrimination & without discrimination?

5. Find the marginal cost functions for each of the following average cost functions.

a. $AC = 1.5Q + 4 + \frac{46}{Q}$

b. $AC = \frac{160}{Q} + 5 - 3Q + 2Q^2$

Practical-7

1. Find the firs order partial derivatives of

a. $z = (9x - 4y)(12x + 2y)$ b. $z = (w - x - y)(3w + 2x - 4y)$

2. Find the firs order partial derivatives of $z = \left(\frac{8x+7y}{5x+2y}\right)^2$

3. Find z_{xx}. a. $z = x^2 + 2xy + y^2$ b. $z = (7x + 3y)^3$

4. Find z_{xy} for $z = 3x^2 + 12xy + 5y^2$

5. Find z_{yx} for $z = wx^2y^4$

Practical-8

1. Find the critical points at which the function may be optimized and determine whether at these points the function is maximized, is minimized, is at an infection point, or is at a saddle point.

a. $z = 3x^2 - xy + 2y^2 - 4x - 7y + 12$

2. For the following function find the critical points and determine if at these points the function is at a relative maximum, relative minimum, infection point or saddle point.

a. $z(x, y) = 3x^3 - 5y^2 - 225x + 70y + 23$

3. Use Lagrange multipliers to optimize the following function subjected to the given constraint and estimate the effect on the value of the objective function from a 1-unit change in the constant of the constraint. $z = 4x^2 - 2xy + 6y^2 \ subject \ to \ x + y = 72$

4. Find the total derivative $\frac{dz}{dx}$ for each of the following functions

a. $z = 6x^2 + 15xy + 3y^2$ b. $z = \frac{9x-7y}{2x+5y}$

5. Find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ of $3y - 12x + 17 = 0$.

Practical-9

1. Find the marginal productivity of the different inputs or factors of production for each of the following production functions Q.

a. $Q = 6x^2 + 3xy + 2y^2$ b. $Q = 0.3K^2 - KL + 2L^2$

2. (a) Assume $\bar{y} = 4$ n Problem (1) and find the MP_x for $x = 5, x = 8$. (b) If the marginal revenue at $\bar{x} = 5, \bar{y} = 4$ is \$3, compute the marginal revenue product for the fifth unit of x.

3. Given a three-sector income determination model in which

$$Y = C + I_0 + G_0, \quad Yd = Y - T, \quad C_0, I_0, G_0, T_0 > 0, \quad 0 < b, t < 1$$

$$C = C_0 + bYd \qquad T = T_0 + tY$$

determine the magnitude and direction of a 1-unit change in (a) government spending, (b) lump-sum taxation, and (c) the tax rate on the equilibrium level of income. In short, perform the comparative-static exercise of determining the government multiplier, the autonomous tax multiplier, and the tax rate multiplier.

3. Given

$$Y = C + I_0 + G_0, \quad Yd = Y - T$$

$$C = C_0 + bYd, \quad T = T_0 + tY$$

where taxation is now a function of income, demonstrate the effect on the equilibrium level of income of a 1-unit change in government expenditure offset by a 1-unit change in autonomous taxation T_0. That is, demonstrate the effect of the balanced-budget multiplier in an economy in which taxes are a positive function of income.

4. Given

$$Y = C + I_0 + G_0 + X_0 - Z \quad T = T_0 + tY$$

$$C = C_0 + bYd, \qquad Z = Z_0 + zYd$$

where all the independent variables are positive and $0 < b, z, t < 1$ Determine the effect on the equilibrium level of income of a 1-unit change in (a) exports, (b) autonomous imports, and (c) autonomous taxation. In short, perform the comparative-static analysis of finding the export, autonomous import, and autonomous taxation multipliers. [Note that $Z = f(Y_d)$].

5. Determine the effect on $\bar{Y}$ of a 1-unit change in the marginal propensity to import z in problem 4.

Practical-10

1.

$$Y = C + I_0 + G_0, \quad Yd = Y - T \quad C_0 = 100, \ I_0 = 90, b = 0.75$$

$$C = C_0 + bYd, \quad T = T_0 + tY, G_0 = 330, T_0 = 240, t = 0.20$$

(a) What is the equilibrium level of income $\bar{Y}$? What is the effect on $\bar{Y}$ of a \$50 increase in (b) government spending and (c) autonomous taxation T_0?

2. (a) If the proportional tax in Problem 1 is increased by 10 percent, what is the effect on $\bar{Y}$?

(b) If the government wants to alter the original marginal tax rate of 20 percent to achieve

$Y_{fe} = 1000$, by how much should it change t?

3. Given $Q = 400 - 8P + 0.05Y$, where $P = 25, Y = 12000$. Find (a) the income elasticity of demand and (b) the growth potential of the product, if income is expanding by 5 percent a year. (c) Comment on the growth potential of the product.

4. A monopolist sells two products x and y for which the demand functions are

$$x = 25 - 0.5P_x \ , \qquad y = 30 - P_y$$

& the combined cost function is $\qquad c = x^2 + 2xy + y^2 + 20$

Find (a) the profit-maximizing level of output for each product, (b) the profit-maximizing price for each product, and (c) the maximum profit.

5. Maximize the following utility functions subject to the given budget constraints,

$u = x^{0.6}y^{0.25}$, given $P_x = 8, P_y = 5, B = 680.$

Practical-11

1. Given a principal P of \$1000 at 6 percent interest i for 3 years, find the future value S when the principal is compounded (a) annually, (b) semiannually, and (c) quarterly.

2. Find the future value of a principal of \$100 at 5 percent for 6 years when compounded (a) annually and (b) continually.

3. Calculate the rate of effective annual interest on \$1000 at 12 percent compounded (a) quarterly and (b) continuously.

4. Determine the interest rate needed to have money double in 10 years under annual compounding.

5. Determine the interest rate needed to have money double in 6 years when compounded semiannually.

Practical-12

1. Find the present value of \$750 to be paid 4 years from now when the prevailing interest rate is 10 percent if interest is compounded (a) annually and (b) semiannually.

2. A firm with sales of 150,000 a year expects to grow by 8 percent a year. Determine the expected level of sales in 6 years.

3. The cost of an average hospital stay was \$500 at the end of 1989. The average cost in 1999 was \$1500. What was the annual rate of increase?

4. Find the future value of a principal of \$2000 compounded semiannually at 12 percent for 3 years, using (a) an exponential function and (b) the equivalent natural exponential function.

5. An animal population goes from 3.5 million in 1997 to 4.97 million in 2001. Express population growth P in terms of a natural exponential function and determine the rate of growth.

Practical-13

1. Find $A + B, A - B$ if $A = \begin{bmatrix} 2 & 6 \\ -4 & 8 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$

2. A firm with five retail stores has 10 $TVs\ t$, 15 $stereos\ s$, 9 $tape\ decks\ d$, and 12 $recorders\ r$ in store 1; $20t, 14s, 8d, and\ 5r$ in store 2; $16t, 8s, 15d, and\ 6r$ in store 3; $25t, 15s, 7d, and\ 16r$ in store 4; and $5t, 12s, 20d, and\ 18r$ in store 5. Express present inventory in matrix form.

3. The parent company in Problem 2 sends out deliveries D to its stores:

$$D = \begin{bmatrix} 4 & 3 & 5 & 2 \\ 0 & 9 & 6 & 1 \\ 5 & 7 & 2 & 6 \\ 12 & 2 & 4 & 8 \\ 9 & 6 & 3 & 5 \end{bmatrix}$$

What is the new level of inventory?

4. A monthly report R on sales for the company in Problem 3 indicates

$$R = \begin{bmatrix} 8 & 12 & 6 & 9 \\ 10 & 11 & 8 & 3 \\ 15 & 6 & 9 & 7 \\ 21 & 14 & 5 & 18 \\ 6 & 11 & 13 & 9 \end{bmatrix}$$

What is the inventory left at the end of the month?

5. Find $A.B$ if $A = \begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 1 \\ 1 & 1 & -1 \end{bmatrix}$ & $B = \begin{bmatrix} 4 & -1 & 2 \\ 1 & -1 & 0 \\ 2 & 1 & 3 \end{bmatrix}$

Practical-14

1. A hamburger chain sells 1000 hamburgers, 600 cheeseburgers, and 1200 milk shakes in a week. The price of a hamburger is 45¢, a cheeseburger 60¢, and a milk shake 50¢. The cost to the chain of a hamburger is 38¢, a cheeseburger 42¢, and a milk shake 32¢. Find the firm's profit for the week, using (a) total concepts and (b) per-unit analysis to prove that matrix multiplication is distributive.

2. Crazy Teddie's sells 700 CDs, 400 cassettes, and 200 CD players each week. The selling price of CDs is $4, cassettes $6, and CD players $150. The cost to the shop is $3.25 for a CD, $4.75 for a cassette, and $125 for a CD player. Find weekly profits by using (a) total and (b) per-unit concepts.

3. Illustrate whether the associative and distributive laws hold for

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 2 \\ 1 & 2 \end{bmatrix} \quad C \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

4. Find $A(BC)$ for

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 3 & -1 & 3 \\ 5 & 2 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 2 \end{bmatrix}, C = \begin{bmatrix} -1 & 4 & 0 \\ 2 & -1 & 0 \\ 1 & 3 & -1 \end{bmatrix}$$

5. Find AB & BA for

$$A = \begin{bmatrix} -2 \\ 5 \\ 3 \end{bmatrix} \ \& \ B = \begin{bmatrix} -1 & 2 & 2 \end{bmatrix}$$

Practical-15

1. Determine the rank of the following matrix

$$\begin{bmatrix} -3 & 6 & 2 \\ 1 & 5 & 4 \\ 4 & -8 & 2 \end{bmatrix}$$

2. The equilibrium condition for two substitute goods is given by

$5P_1 - 2P_2 = 15, \ \ -P_1 + 8P_2 = 16$

Find the equilibrium prices.

3. The equilibrium condition for three related markets is given by

$$11P_1 - P_2 - P_3 = 31, \quad -P_1 + 6P_2 - 2P_3 = 26, \qquad -P_1 - 2P_2 + 7P_3 = 24$$

Find the equilibrium price for each market.

4. Given $Y = C + I_{0,}$ where $C = C_0 + bY$. Use matrix inversion to find the equilibrium level of Y and C.

5. Determine the total demand x for industries 1, 2, and 3, given the matrix of technical coefficients A and the final demand vector B.

$$A = \begin{bmatrix} 0.3 & 0.4 & 0.1 \\ 0.5 & 0.2 & 0.6 \\ 0.1 & 0.3 & 0.1 \end{bmatrix} \quad \& \ B = \begin{bmatrix} 20 \\ 10 \\ 30 \end{bmatrix}$$

Practical-16

1. Optimize the following function, using (a) Cramer's rule for the first-order condition and (b) the Hessian for the second-order condition:

$$y = 3x_1^2 - 5x_1 - x_1x_2 + 6x_2^2 - 4x_2 + 2x_2x_3 + 4x_3^2 + 2x_3 - 3x_1x_3$$

2. A firm produces two goods in pure competition and has the following total revenue and total cost functions:

$$TR = 15Q_1 + 18Q_2 \qquad\qquad TC = 2Q_1^2 + 2Q_1Q_2 + 3Q_2^2$$

The two goods are technically related in production, since the marginal cost of one is dependent on the level of output of the other. Maximize profits for the firm, using (a) Cramer's rule for the first-order condition and (b) the Hessian for the second-order condition.

3. Maximize profits for a producer of two substitute goods, given

$$P_1 = 130 - 4Q_1 - Q_2 \qquad P_2 = 160 - 2Q_1 - 5Q_2 \qquad TC = 2Q_1^2 + 2Q_1Q_2 + 4Q_2^2$$

Use (a) Cramer's rule for the first-order condition and (b) the Hessian for the second-order condition.

4. Maximize utility $u = 2xy$ subject to a budget constraint equal to $3x + 4y = 90$ by (a) finding the critical values $\bar{x}, \bar{y}$ and $\bar{\lambda}$ and (b) using the bordered Hessian $\bar{H}$ to test the second-order condition.

5. Minimize a firm's total costs $c = 45x^2 + 90xy + 90y^2$ when the firm has to meet a production quota g equal to $2x + 3y = 60$ by (a) finding the critical values and (b) using the bordered Hessian to test the second-order conditions.

Practical-17

1. Assume a two-sector income determination model where consumption depends on income and investment is autonomous, so that

$$C = bY, I = I_0, 0 < b < 1$$

And equilibrium occurs when $Y = C + I$. Solve for the equilibrium level of income Y^*explicitly.

2. Assume a two-sector income determination model where consumption depends on income and investment is autonomous, so that

$$C = bY, I = I_0, 0 < b < 1$$

And equilibrium occurs when $Y = C + I$. Use comparative statics to estimate the effect on Y^* of a change in autonomous investment I_0.

3. Assume a two-sector income determination model where consumption depends on income and investment is autonomous, so that

$$C = bY, I = I_0, 0 < b < 1$$

And equilibrium occurs when $Y = C + I$. Find the same comparative-static derivative from the implicit function.

4. Assume a two-sector income determination model where consumption depends on income and investment is autonomous, so that

$$C = bY, I = I_0, 0 < b < 1$$

And equilibrium occurs when $Y = C + I$. Evaluate the effect on Y^* of a change in the marginal propensity to consume b explicitly and implicitly.

5. Assume a two-sector income determination model expressed in general functions:

$$C = C(Y), \ \ I = I_0$$

with equilibrium when $Y = C + I$. Determine the implicit function for the equilibrium level of income Y^*.

Practical-18

1. Given the income determination model $Y = C + I_0 + G_0 + X_0 - Z \qquad C = C_0 + bY$

$Z = Z_0 + zY$ where $X =$ exports, $Z =$ imports, and a zero subscript indicates an exogenously fixed variable, (a) express the system of equations as both general and specific implicit functions. (b) Express in matrix form the total derivatives of both the general and the specific functions with respect to exports X_0.

2. Given the income determination model $Y = C + I_0 + G_0 + X_0 - Z \qquad C = C_0 + bY$

$Z = Z_0 + zY$ where $X =$ exports, $Z =$ imports, and a zero subscript indicates an exogenously fixed variable, Express in matrix form the total derivatives of both the general and the specific functions with respect to exports X_0. Then find and sign $\frac{\partial \bar{Y}}{\partial X_0}, \frac{\partial \bar{C}}{\partial X_0}, \frac{\partial \bar{Z}}{\partial X_0}$.

3. Using the model from Problem 1 express in matrix form the total derivatives of the specific functions with respect to the marginal propensity to consume b.

4. Using Problem 3 find and sign $\frac{\partial \bar{Y}}{\partial b}, \frac{\partial \bar{C}}{\partial b}, \frac{\partial \bar{Z}}{\partial b}$.

5. Continuing with the model from Problem 1 express in matrix form the total derivatives of the specific functions with respect to the marginal propensity to import z.

Practical-19

1. Evaluate $\int 10x(x^2+3)^4 dx$

2. Evaluate $\int x^4(2x^5-5)^4 dx$

3. Evaluate $\int 15x(x+4)^{3/2} dx$

4. The rate of net investment is $I = 40t^{3/5}$, and capital stock at $t = 0$ is 75. Find the capital function K.

5. Marginal cost is given by $MC = \frac{dTC}{dQ} = 25 + 30Q - 9Q^2$. Fixed cost is 55. Find the (a) total cost, (b) average cost, and (c) variable cost functions.

Practical-20

1. Evaluate $\int_1^3 \frac{4x}{(x+2)^3} dx$

2. (a) Draw the graphs of the following functions, and (b) evaluate the area between the curves over the stated interval:

$y_1 = 7 - x \qquad y_2 = 4x - x^2 \quad from\, x = 1 \quad to \quad x = 4.$

3. (a) Specify why the integral given below is improper and (b) test for convergence. Evaluate where possible.

$$\int_1^\infty \frac{2x\,dx}{(x^2+1)^2}$$

4. Given the demand function $P = 45 - 0.5Q$, find the consumers' surplus CS when $P_0 = 32.5$ and $Q_0 = 25$.

5. Under a monopoly, the quantity sold and market price are determined by the demand function. If the demand function for a profit-maximizing monopolist is $P = 274 - Q^2$ and $MC = 4 + 3Q$, find the consumers' surplus.

Practical-21

1. Solve $\frac{dy}{dt} + 4y = -20, y(0) = 10\text{e}$

2. Solve $\frac{dy}{dt} + 4ty = 6t$

3. Solve $(4y + 8t^2)dy + (16yt - 3)dt = 0$

4. Solve $(7y + 4t^2)dy + 4ty\,dt = 0$

5. Solve $y^3 t\,dy + \frac{1}{2}y^4 dt = 0$

Practical-22

1. Find the demand function $Q = f(P)$ if point elasticity $\in$ is -1 for all $P > 0$.

2. Find the demand function $Q = f(P)$ if $\in = -\frac{(5P+2P^2)}{Q}$ and $Q = 500,\ when\ P = 10$.

3.(a) Construct a phase diagram for the following nonlinear differential equation and test the dynamic stability using (b) the arrows of motion, (c) the slope of the phase line, and (d) the derivative test

$$\dot{y} = 3y^2 - 18y$$

4. A change in the rate of investment will affect both aggregate demand and the productive capability of an economy. The Domar model seeks to find the time path along which an economy can grow while maintaining full utilization of its productive capacity. If the marginal propensity to save s and the marginal capital-output ratio k are constant, find the investment function needed for the desired growth.

5. Assume that the demand for money is for transaction purposes only. Thus, $M_d = kP(t)Q$ where k is constant, P is the price level, and Q is real output. Assume $M_s = M_d$ and is exogenously determined by monetary authorities. If inflation or the rate of change of prices is proportional to excess demand for goods in society and, from Walras' law, an excess demand for goods is the same thing as an excess supply of money, so that

$$\frac{dP(t)}{dt} = b(M_s - M_d)$$

find the stability conditions, when real output Q is constant.

Practical-23

1. (a) Solve the difference equation given below; (b) check your answer, using $t = 0$ and $t = 1$; and (c) comment on the nature of the time path.

$$y_t = 6y_{t-1}$$

2. (a) Solve the difference equation given below; (b) check your answer, using $t = 0$ and $t = 1$; and (c) comment on the nature of the time path.

$$5y_t + 2y_{t-1} - 140 = 0,\ \ y_0 = 30$$

3. (a) Solve the difference equation given below; (b) check your answer, using $t = 0$ and $t = 1$; and (c) comment on the nature of the time path.

$$y_t - y_{t-1} = 17$$

4. (a) Solve the difference equation given below; (b) check your answer, using $t = 0$ and $t = 1$; and (c) comment on the nature of the time path.

$$g_t = g_{t-1} - 25 = 0,\ \ g_0 = 40$$

5. (a) Solve the difference equation given below; (b) check your answer, using $t = 0$ and $t = 1$; and (c) comment on the nature of the time path.

$$\Delta y_t = y_t + 13,\ \ y_0 = 45$$

Practical-24

1. Given the data below, (a) find the time path of national income Y_t; (b) check your answer, using $t = 0$ and $t = 1$; and (c) comment on the stability of the time path.

$$C_t = 90 + 0.8Y_{t-1}, \qquad I_t = 50, \qquad Y_0 = 1200$$

2. Given the data below, (a) find the time path of national income Y_t; (b) check your answer, using $t = 0$ and $t = 1$; and (c) comment on the stability of the time path.

$$C_t = 400 + 0.6Y_t + 0.35Y_{t-1}, \qquad I_t = 240 + 0.15Y_{t-1}, \qquad Y_0 = 7000$$

3. For the data given below, determine (a) the market price P_t in any time period, (b) the equilibrium price P_e, and (c) the stability of the time path.

$Q_{dt} = 180 - 0.75P_t, \quad Q_{st} = -30 - 0.3P_{t-1}\,, \; P_0 = 220$

4. For the following data, find (a) the level of income Y_t for any period and (b) the warranted rate of growth. $\quad I_t = 2.66(y_t - y_{t-1}), \;\; S_t = 0.16Y_t, \;\; Y_0 = 9000$

5. (a) Construct a phase diagram for the nonlinear difference equation below, (b) use it to test for dynamic stability, and (c) confirm your results with the derivative test.

$$y_t = y_{t-1}^3$$